Función Constante:

La derivada de una función constante es siempre igual a 0.

y= f(x) = 4 **sol**: $y' = 0$
y= f(x) = -1500 **sol**: $y' = 0$
y= f(x) = 0 **sol**: $y' = 0$

Función Potencial:

$y = f^n(x) \rightarrow y' = n \cdot f(x)^{n-1} \cdot f'(x)$

$y = x$ **sol**: $y' = 1$
$y = 4x^7$ **sol**: $y' = 28x^6 \cdot 1 = 28x^6$
$y = x^3$ **sol**: $y' = 3x^2 \cdot 1 = 3x^2$
$y = (2x+1)^3$ **sol**: $y' = 3(2x+1)^2 \cdot 2 = 6(2x+1)^2 \cdot$
$y = (3x^2 - 4x)^5$ **sol**: $y' = 5(3x^2-4x)^4 \cdot (6x-4)$

Concepto de derivada:

- $y = x \rightarrow y' = 1$

sol: $y' = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)-x}{h} = \lim_{h\to 0} \frac{h}{h} = \left[\frac{0}{0}\right] \quad y' = 1$

- $y = 4x^2 \rightarrow y' = 8x$

sol: $y' = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{4(x+h)^2-4x^2}{h} = \lim_{h\to 0} \frac{4x^2+4h^2+8xh-4x^2}{h} = \lim_{h\to 0} \frac{4h^2+8xh}{h} = \left[\frac{0}{0}\right] \quad \lim_{h\to 0} \frac{\not{h}(4h+8x)}{\not{h}} = \lim_{h\to 0}(4h+8x) = 8x$

Función Irracional:

$y = \sqrt[n]{f(x)} \rightarrow y' = \frac{f'(x)}{n\sqrt[n]{f^{n-1}(x)}}$

$y = \sqrt{x}$ **sol**: $y' = \frac{1}{2\sqrt{x}}$
$y = \sqrt[3]{x^2}$ **sol**: $y' = \frac{2x}{3\sqrt[3]{(x^2)^2}} = \frac{2x}{3\sqrt[3]{x^4}}$
$y = \sqrt[5]{(x^3-2)^4}$ **sol**: $y' = \frac{4(x^3-2)^3 \cdot (3x^2-0)}{5\sqrt[5]{((x^3-2)^4)^4}} = \frac{4(x^3-2)^3 \cdot (3x^2)}{5\sqrt[5]{(x^3-2)^{16}}} = \frac{12x^2}{5\sqrt[5]{x^3-2}}$

Puedes realizar también las derivadas transformando la función irracional en potencial

$\mathbf{y = \sqrt[5]{(x^3-2)^4} \rightarrow y = (x^3-2)^{\frac{4}{5}}}$

$\mathbf{y' = \frac{4}{5}(x^3-2)^{\frac{4}{5}-1} \cdot (3x^2) = \frac{4}{5}(x^3-2)^{\frac{-1}{5}} \cdot (3x^2) = \frac{12x^2}{5\sqrt[5]{x^3-2}}}$

Concepto de derivada:

- y= $\sqrt{x}$ → y´= $\frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{\sqrt{x}}{2x}$

sol: y´= $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} = \left[\frac{0}{0}\right]$

$$\lim_{h\to 0} \frac{(\sqrt{x+h}-\sqrt{x})}{h} \cdot \frac{(\sqrt{x+h}+\sqrt{x})}{(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0} \frac{(x+h)-(x)}{h\cdot(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0} \frac{h}{h\cdot(\sqrt{x+h}+\sqrt{x})} = \left[\frac{0}{0}\right]$$

$$\lim_{h\to 0} \frac{1}{(\sqrt{x+h}+\sqrt{x})} = \frac{1}{\sqrt{x}+\sqrt{x}} = \frac{1}{2\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{\sqrt{x}}{2x}$$

Función Exponencial:

y=a$^{f(x)}$ → y´= a$^{f(x)}$·Lna·f ´(x)

y = $\mathbf{e^x}$ — **sol: y´=** $\mathbf{e^x \cdot lne \cdot 1 = e^x}$

y = $\mathbf{3^x}$ — **sol: y´=** $\mathbf{3^x \cdot ln3 \cdot 1 = 3^x \cdot ln3}$

y = $\mathbf{5^{x^3-2x}}$ — **sol: y´=** $\mathbf{5^{x^3-2x} \cdot (ln5) \cdot (3x^2-2)}$

y = $\mathbf{e^{5x^3-7}}$ — **sol: y´ =** $\mathbf{e^{5x^3-7} \cdot (lne) \cdot (15x^2-0) = e^{5x^3-7} \cdot (15x^2)}$

Función Logarítmica:

y = log$_a$ f(x) → y´= $\frac{f'(x)}{f(x)\cdot Lna}$

y = lnx = $\log_e x$ — **Sol**: y´= $\frac{1}{x\cdot lne} = \frac{1}{x}$

y = $\ln(3x^2-6)$ = $\log_e(3x^2-6)$ — **Sol**: y´= $\frac{6x-0}{(3x^2-6)\cdot lne} = \frac{6x}{3x^2-6}$

y = $\log_3(5x^4-2)$ — **Sol**: y´= $\frac{20x^3-0}{(5x^4-2)\cdot ln3} = \frac{20x^3}{(5x^4-2)\cdot ln3}$

y = $\log_7(7x^3-5x)$ — **Sol**: y´= $\frac{21x^2-5}{(7x^3-5x)\cdot ln7}$

y = $\log_5(3x^2-8x)^6$ — **Sol**: y´ = $\frac{6(3x^2-8x)^5\cdot(6x-8)}{(3x^2-8x)^6\cdot ln5} = \frac{36x-48}{(3x^2-8x)\cdot ln5}$

Operaciones con derivadas

- Producto: [f(x)·g(x)] = f ’(x)·g(x)+f(x)·g’(x)

y = $3^{x^3+2} \cdot (3x^4-5x^2)$ — **Sol**: y´ = $3^{x^3+2} \cdot (3x^2)\cdot(ln3)\cdot(3x^4-5x^2) + 3^{x^3+2}\cdot(12x^3-10x)$

y = $\log_3(x^2-3) \cdot (\sqrt[3]{2x})$ — **Sol**: y´ = $\frac{2x}{(x^2-3)\cdot ln3} \cdot (\sqrt[3]{2x}) + \log_3(x^2-3) \cdot \frac{2}{3\sqrt[3]{(2x)^2}}$

y = $\log_3(x^3-2x) \cdot 3^{2x-1}$ — **Sol**: y´ = $\left(\frac{3x^2-2}{(x^3-2x)\cdot ln3}\right) \cdot 3^{2x-1} + \log_3(x^3-2x)\cdot(3^{2x-1}\cdot 2 \cdot ln3)$

Concepto de derivada:

- y= $x^2 \cdot (x-1) \rightarrow$ y´= 2x· $(x-1) + 1 \cdot x^2 = 2x^2 - 2x + x^2 = 3x^2 - 2x$

sol: $y' = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2\cdot(x+h-1) - x^2\cdot(x-1)}{h} = \lim_{h\to 0} \frac{(x^2+h^2+2xh)(x+h-1)-(x^3-x^2)}{h} =$

$$\lim_{h\to 0} \frac{x^3+xh^2+2x^2h+x^2h+h^3+2xh^2-x^2-h^2-2xh-x^3+x^2}{h} = \lim_{h\to 0} \frac{3xh^2+3x^2h+h^3-h^2-2xh}{h} = \left[\frac{0}{0}\right]$$

$$\lim_{h\to 0} \frac{\cancel{h}(3xh+3x^2+h^2-h-2x)}{\cancel{h}} = \mathbf{3x^2 - 2x}$$

- Cociente: $\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{g^2(x)}$

y = $\frac{5^{3x}}{(3x^2+4)}$

Sol: y´ = $\frac{(5^{3x}\cdot 3\cdot ln5)\cdot(3x^2+4)-(6x)\cdot 5^{3x}}{(3x^2+4)^2}$

y = $\frac{\ln(x^3-7x)}{(x^5-2)}$

Sol: y´ = $\frac{\frac{3x^2-7}{(x^3-7x)}\cdot(x^5-2) - (5x^4)\cdot\ln(x^3-7x)}{(x^5-2)^2}$

y = $\frac{3^{2x-1}}{\log_3(x^3-2x)\cdot}$

Sol: y´ = $\frac{(3^{2x-1}\cdot 2\cdot ln3)\cdot\left(\log_3(x^3-2x) - \left(\frac{3x^2-2}{(x^3-2x)\cdot ln3}\right)\cdot 3^{2x-1}\right)}{(\log_3(x^3-2x)\cdot)^2}$

Concepto de derivada:

- y= $\frac{x^2}{x-1} \rightarrow$ y´= $\frac{2x\cdot(x-1)-1\cdot(x^2)}{(x-1)^2} = \frac{2x^2-2x-x^2}{(x-1)^2} = \mathbf{\frac{x^2-2x}{(x-1)^2}}$

sol: $y' = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{(x+h)^2}{(x+h-1)} - \frac{x^2}{x-1}}{h} = \lim_{h\to 0} \frac{\frac{x^2+h^2+2xh}{(x+h-1)} - \frac{x^2}{x-1}}{h} = \lim_{h\to 0} \frac{\frac{(x^2+h^2+2xh)(x-1)-x^2(x+h-1)}{(x+h-1)(x-1)}}{h} =$

$$\lim_{h\to 0} \frac{\frac{x^3+xh^2+2x^2h-x^2-h^2-2xh-x^3-x^2h+x^2}{(x+h-1)(x-1)}}{h} = \lim_{h\to 0} \frac{\frac{xh^2+x^2h-h^2-2xh}{(x+h-1)(x-1)}}{h} = \lim_{h\to 0} \frac{xh^2+x^2h-h^2-2xh}{h(x+h-1)(x-1)} = \left[\frac{0}{0}\right]$$

$$\lim_{h\to 0} \frac{\cancel{h}(xh+x^2-h-2x)}{\cancel{h}(x+h-1)(x-1)} = \lim_{h\to 0} \frac{(xh+x^2-h-2x)}{(x+h-1)(x-1)} = \mathbf{\frac{x^2-2x}{(x-1)^2}}$$

Ejercicios

1. $y=(4x^3-5)^4+3x$ Sol: $y'=4(4x^3-5)^3\cdot(12x^2)+3$

2. $y=\sqrt[3]{x}(3x^2+6x)$ Sol: $y'=\frac{1}{3\sqrt[3]{x^2}}\cdot(3x^2+6x)+\sqrt[3]{x}\cdot(6x+6)$

3. $y=\sqrt{\frac{3x^2+2}{x}}$ Sol: $y'=\frac{1}{2\sqrt{\frac{3x^2+2}{x}}}\cdot\frac{6x\cdot x-1\cdot(3x^2+2)}{x^2}$

4. $y=\ln(x^3-2x)^3$ Sol: $y'=\frac{1}{(x^3-2x)^3\cdot Lne}\cdot 3(x^3-2x)^2\cdot(3x^2-2)$

5. $y=\ln^2(x^3-2x)^4$ Sol: $y'=2(\ln(x^3-2x)^4)\cdot\frac{4(x^3-2x)^3\cdot(3x^2-2)}{(x^3-2x)^4\cdot Lne}$

6. $y=\left(6x^3+\sqrt{x^3}\right)\ln x$ Sol: $y'=\left(18x^2+\frac{3x^2}{2\sqrt{x^3}}\right)\cdot(\ln x)+\left(6x^3+\sqrt{x^3}\right)\cdot\frac{1}{x}$

7. $y=(x^3-5x+2)^4\cdot\ln x$ Sol: $y'=4(x^3-5x+2)^3\cdot(3x^2-5)\cdot(\ln x)+(x^3-5x+2)^4\cdot\frac{1}{x}$

8. $y=(4x^3-\sqrt{x})^3\ln x^4$ Sol: $y'=3\left(4x^3-\sqrt{x}\right)^2\cdot\left(12x^2-\frac{1}{2\sqrt{x}}\right)\cdot(\ln x^4)+\left(4x^3-\sqrt{x}\right)^3\cdot\frac{4x^3}{x^4}$

9. $y=\ln^3(x-3)$ Sol: $y'=3(\ln(x-3))^2\cdot\frac{1}{x-3}$

10. $y=\frac{3x^2-2x}{x+1}$ Sol: $y'=\frac{(6x-2)(x+1)-1(3x^2-2x)}{(x+1)^2}$

11. $y=2^{x^2-1}\ln(x^3-2x)$ Sol: $y'=\left(2^{x^2-1}\cdot 2x\cdot \ln 2\right)\cdot(\ln(x^3-2x))+2^{x^2-1}\cdot\frac{(3x^2-2)}{x^3-2x}$

12. $y=\ln\left(\frac{3x+2}{x^2}\right)^2$ Sol: $y'=\frac{1}{\left(\frac{3x+2}{x^2}\right)^2\cdot(\ln e)}\cdot 2\left(\frac{3x+2}{x^2}\right)\cdot\frac{3\cdot x^2-2x\cdot(3x+2)}{(x^2)^2}$

13. $y=e^{4x^2+2x}$ Sol: $y'=e^{4x^2+2x}\cdot(8x+2)\cdot(\ln e)$

14. $y=\frac{e^{2x}+1}{e^{2x}-1}$ Sol: $y'=\frac{(e^{2x}\cdot 2\cdot\ln e)\cdot(e^{2x}-1)-(e^{2x}\cdot 2\cdot\ln e)\cdot(e^{2x}+1)}{(e^{2x}-1)^2}$

15. $y=(3x^2+5x)\ln(x-1)$ Sol: $y'=(6x+5)\cdot(\ln(x-1))+(3x^2+5x)\cdot\frac{1}{x-1}$

16. $y=\sqrt[4]{3x^3+2x}$ Sol: $y'=\frac{9x^2+2}{4\sqrt[4]{(3x^3+2x)^3}}$

17. $y=\frac{2}{x^3+2x^4}$ Sol: $y'=\frac{0\cdot(x^3+2x^4)-2\cdot(3x^2+8x^3)}{(x^3+2x^4)^2}=\frac{-2(3x^2+8x^3)}{(x^3+2x^4)^2}$

18. $y = 3x^4\left(x^3+2\right)^5$ Sol: $y' = 12x^3 \cdot (x^3+2)^5 + 5(x^3+2)^4 \cdot (3x^2) \cdot 3x^4$

19. $y = 7^{\ln(x^2-5x)}$ Sol: $y' = 7^{\ln(x^2-5x)} \cdot \frac{(2x-5)}{(x^2-5x)\cdot(\ln e)} \cdot (\ln 7)$

20. $y = \frac{4x+3}{x-3}$ Sol: $y' = \frac{4\cdot(x-3)-1\cdot(4x+3)}{(x-3)^2} = \frac{-6}{(x-3)^2}$

21. $y = \left(x^4+3\right)\ln(2x+1)$ Sol: $y' = (4x^3)\cdot\left(\ln(2x+1)\right) + (x^4+3)\cdot\frac{2}{(2x+1)}$

22. $y = \frac{4}{\sqrt{x^3+5}}$ Sol: $y' = \frac{0\cdot\sqrt{x^3+5}-4\cdot\frac{3x^2}{2\sqrt{x^3+5}}}{\left(\sqrt{x^3+5}\right)^2} = \frac{-6x^2}{\sqrt{(x^3+5)^3}}$

23. $y = \frac{2^x+1}{2^x}$ Sol: $y' = \frac{(2^x\cdot 1\cdot(\ln 2))\cdot 2^x-\left(2^x\cdot 1\cdot(\ln 2)\right)\cdot(2^x+1)}{(2^x)^2}$

24. $y = \frac{2}{e^{3x}}$ Sol: $y' = \frac{0\cdot e^{3x}-(e^{3x}\cdot 3\cdot\ln e)\cdot 2}{\left(e^{3x}\right)^2} = \frac{-6e^{3x}}{\left(e^{3x}\right)^2} = \frac{-6}{e^{3x}}$

25. $y = \ln\sqrt{\frac{x+2}{x-1}}$ Sol: $y' = \frac{\frac{\frac{1\cdot(x-1)-1\cdot(x+2)}{(x-1)^2}}{2\sqrt{\frac{x+2}{x-1}}}}{\sqrt{\frac{x+2}{x-1}}\cdot \ln e} = \frac{\frac{-3}{(x-1)^2}}{2\cdot\left(\frac{x+2}{x-1}\right)} = \frac{-3}{2(x-1)(x+2)} = \frac{-3}{2x^2+2x-4}$

26. $y = \log_3\left(x^3-2x-3\right)^4$ Sol: $y' = \frac{4(x^3-2x-3)^3\cdot(3x^2-2)}{(x^3-2x-3)^4\cdot(\ln 3)} = \frac{12x^2-8}{(x^3-2x-3)\cdot(\ln 3)}$

27. $y = e^{3x^2-2}\sqrt{x^2-6}$ Sol: $y' = \left(e^{3x^2-2}\cdot(6x)\right)\cdot\left(\sqrt{x^2-6}\right) + \left(\frac{2x}{2\sqrt{x^2-6}}\right)\cdot\left(e^{3x^2-2}\right)$

28. $y = (\log_3(3x^3-2x-1)^4)^5$ Sol: $y' = 5(\log_3(3x^3-2x-1)^4)^4 \cdot \frac{4(3x^3-2x-1)^3\cdot(9x^2-2)}{(3x^3-2x-1)^4\cdot(\ln 3)}$